Control of Turing pattern formation by delayed feedback

Qian Shu Li* and Lin Ji

School of Science, Beijing Institute of Technology, Beijing 100081, China

(Received 24 October 2003; revised manuscript received 7 January 2004; published 27 April 2004)

The effect of the global delayed feedback technique on Turing pattern formation is investigated in the modified Lengyel-Epstein two-variable model. Feedback intensity, delay time, and feedback-imposing time (the period of time that feedback is present in the system) are all found to be of significant influence on Turing pattern formation time. Under appropriate parameter settings, delayed feedback could suppress or induce the Turing pattern if the feedback intensity is strong enough.

DOI: 10.1103/PhysRevE.69.046205 PACS number(s): 89.75.Kd, 82.40.Ck, 47.54.+r

I. INTRODUCTION

The coupling of reaction and diffusion processes in nonlinear systems far from the thermodynamic equilibrium can produce various spatiotemporal patterns, such as stationary concentration patterns (Turing patterns), rotating spiral waves, and some spatiotemporal chaos patterns. These patterns exhibit rich dynamic behaviors, which might be undesirable under many circumstances. To control these patterns, there are mainly two ways: exerting artificial external perturbation or utilizing acting force generated by the system itself, i.e., feedback. Feedback control of the pattern dynamics in spatially extended system has been intensively investigated in recent years [1,2]. It cannot only stabilize spatiotemporal chaotic [3–5] and oscillating [6] states, but also induce chaotic, regular spatiotemporal patterns, and clusters in some other situations [7–9].

Turing pattern formation, due to its potential connection with the biological morphogenesis, has been extensively studied both theoretically and experimentally [10,11]. Its dynamic behavior under external fluctuation is a subject of growing interest [12–17]. The Turing pattern could be suppressed by constant or periodically changing illumination [13,14]. However, it could also be induced by spatial noise after its disappearance at high external light intensity [15,16]. The investigation of Turing pattern dynamics under spatiotemporal forcing shows that it is possible to induce new, generic dynamical behaviors from strictly spatial resonance [17]. But the effects of feedback strategies on the Turing pattern remain almost unknown. In this paper, we report the numerical simulation results of Turing pattern formation under global delayed feedback control. The effects of delay time, feedback strength, and feedback-imposing time are systematically investigated.

II. MODEL

We use the modified Lengyel-Epstein two-variable model [18], which includes the illumination effect [19] for the photosensitive chlorine dioxide-iodine-malonic acid (CDIMA) reaction [20]

$$\partial_t u = a - u - 4\frac{uv}{1+u^2} - \phi + \nabla^2 u,$$

$$\partial_t v = \sigma\left[b\left(u - \frac{uv}{1+u^2} + \phi\right) + e\nabla^2 v\right]. \qquad (1)$$

The variables u and v represent the dimensionless concentrations of activator and inhibitor, respectively. a, b, e, and σ are dimensionless parameters; the feedback is introduced by external illumination $\phi=\phi(t)=\phi_0-P[v(t-\tau)-v_0]$, where P is the feedback intensity of the control strategy, function $v(t-\tau)$ denotes the delayed local concentration of an arbitrarily chosen spot (center spot in this simulation), τ is the effective delay time, ϕ_0 is the reference light intensity, and v_0 is a constant local concentration value of the stationary Turing pattern obtained under constant illumination of ϕ_0 when the delayed feedback effect is absent. In this simulation other parameters are fixed at $a=36$, $b=2.5$, $e=1.2$, and $\sigma=9$ in order to ensure that the change of external light intensity could produce different kinds of Turing pattern [14]. Integration of Eq. (1) is performed on a 100×100 square lattice, utilizing Heun algorithm with fixed step of 0.005 t.u. (time unit), no-flux boundary condition and random initial condition. All the data in this work are the average of 15 independent runs.

III. RESULTS AND DISCUSSION

We first investigate the effect of feedback intensity P on Turing pattern formation under different feedback parameter settings in Fig. 1. Feedback is proved to have a prominent effect on Turing pattern formation time (PFT). Because the parameter setting chosen here is below the Turing line in parameter space (in the region where uniform steady state cannot appear) [14], PFT is defined as the pattern evolution time after which the variation of the monitored local concentration is less than 10^{-3} of its value. From Fig. 1(a) we can see that when τ is set at different values, PFT presents different trends with the increase of P. PFT can be greatly and rapidly lengthened when τ is set at some value [e.g., $\tau=2.0$ t.u. in Fig. 1(a)]. As P accumulates to some extent [e.g., τ=2.0 t.u., $P>0.009$ in Fig. 1(a)], we cannot find PFT (pattern evolution time is set above 5000 t.u.to make sure the

*Email address: qsli@bit.edu.cn. Fax: +86-10-68912665

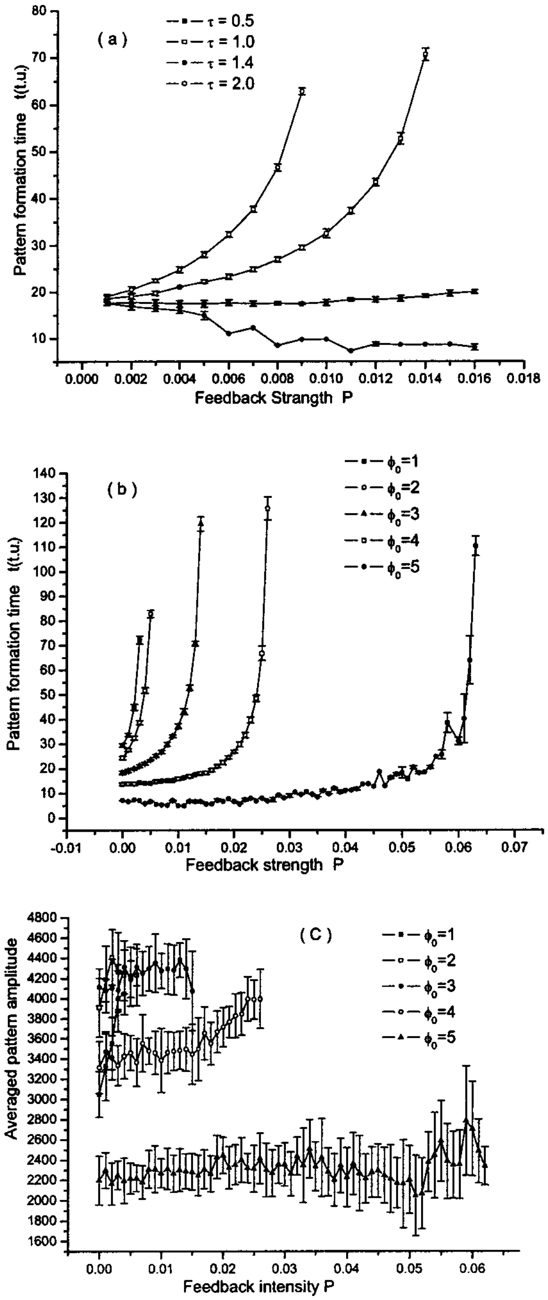

FIG. 1. Effect of feedback intensity on Turing pattern. (a) The variation of pattern formation time under different τ, $\phi_0=3$. (b) The variation of pattern formation time under different ϕ_0, $\tau=1$ (t.u.). (c) The variation of averaged pattern amplitude time under different ϕ_0, $\tau=1$ (t.u.). Error bars represent standard deviation.

system is fully developed) because the stationary patterns are suppressed and bulk oscillations ensue. At other τ values, PFT is hardly lengthened [e.g., $\tau=0.5$ t.u. in Fig. 1(a)] or even shortened [e.g., $\tau=1.4$ t.u. in Fig. 1(a)]. This means τ also has significant control effect on PFT. In order to intensively investigate the pattern suppression effect, we choose $\tau=1.0$ t.u. to study the influence of different ϕ_0 values. From the corresponding results shown in Fig. 1(b), we can see that the larger ϕ_0, the larger the P value that can produce pattern suppression. In order to compare the size of feedback with

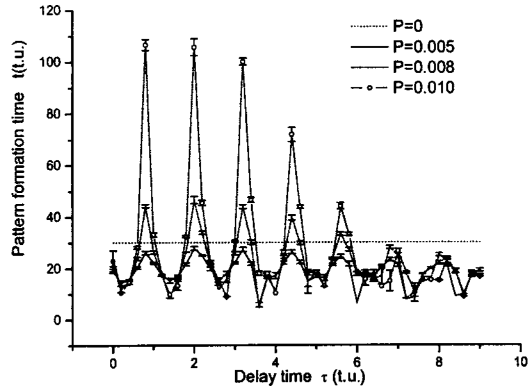

FIG. 2. The variation of pattern formation time with delay time. $\phi_0=3$. Dotted line: $P=0$. Solid line: $P=0.005$. Dash line: $P=0.008$. Error bars represent standard deviation.

the reference light intensity, the influence of feedback on pattern amplitude is investigated in Fig. 1(c). We find that under the amount of feedback which can effectively control Turing pattern formation, the amplitude of the pattern changes little. The main difference of pattern amplitude is brought by the reference light intensity ϕ_0.

The impact of delay time on PFT can be seen in Fig. 2. PFT varies periodically with the increase of delay time. The intervals between neighbor peaks (valleys) are nearly the same, which almost coincides with the period of transient oscillation before stationary pattern formation in the feedback-absent system. The reference dotted line in Fig. 2 represents the case that the external feedback is absent ($P=0$). We find PFT can either be lengthened (above $P=0$) or shortened (below $P=0$). P and τ together determine whether the Turing pattern could be suppressed, but P is the dominant factor between them. When P is not big enough ($P=0.005$), system PFT can never be lengthened no matter what τ is. Only when P is big enough ($P=0.008$ or 0.010) and τ is set at some appropriate value can PFT of the system be lengthened and finally the pattern be suppressed. This phenomenon suggests the system may possess an intrinsic *response time*. Only after this period of time can the system respond to the influence of external illumination.

The abovementioned phenomena may be explained in the transient oscillation process of Turing pattern formation. On the one hand, the internal nonlinear feedback mechanism of the system is prone to stabilize itself in a regular Turing pattern. On the other hand, because the delay can effectively modify the phase difference between the external feedback signal and the system internal nonlinear feedback signal, the external feedback exerted here can either strengthen or weaken the internal feedback. When the two feedbacks are in phase, the internal nonlinear feedback mechanism is strengthened, they will cooperate to stabilize the system, and the PFT is shortened with the increase of P [e.g., $\tau=1.4$ t.u. in Fig. 1(a)]. When the two feedbacks are antiphase, the internal nonlinear feedback mechanism is weakened, and they will compete. With the increase of P, the internal nonlinear feedback mechanism has to take longer time to dominate in the competition and PFT is lengthened [e.g., $\tau=1.0$ and 2.0 t.u. in Fig. 1(a)]. When P reaches a

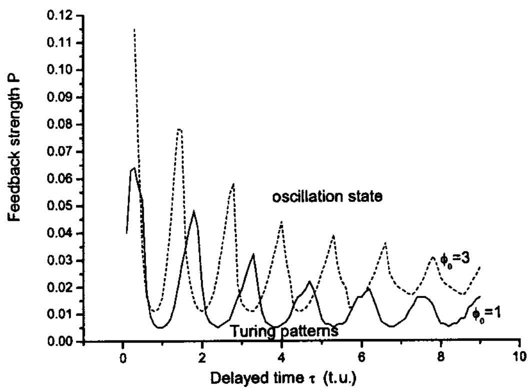

FIG. 3. Regions of Turing pattern and bulk oscillation state in the delayed feedback controlled Turing system. The solid line ($\phi_0=1$) or the dash line ($\phi_0=3$) depicts the boundary between Turing pattern and bulk oscillation state.

certain level, the internal nonlinear feedback mechanism can no longer counterbalance the external feedback, oscillation state dominates and the patterns are suppressed. The larger the reference light intensity ϕ_0 is, the stronger the system self-adjust ability is. Thus, the system will require stronger external feedback to counterbalance its internal nonlinear feedback [Fig. 1(b)]. Changing τ will produce different phase differences between the internal and external feedbacks, and PFT will change accordingly (Fig. 2).

The investigation on the effect of part time feedback (i.e., the feedback is imposed from the beginning of pattern evolution but stopped in middle of the process) shows the control of PFT can also be achieved by changing the feedback-imposing time. Before reaching the point where the full control effect of pattern formation is realized, applying the feedback for a longer time has a larger effect. But after that, continuing to apply the perturbation will produce no further effect.

Figure 3 shows the bifurcation diagram of delayed feedback-induced Turing pattern suppression. We can see the boundary between Turing pattern and bulk oscillation is *spiky* and displays a clear, almost periodic structure. The intervals between neighbor peaks (valleys) are also about the same as the period of transient oscillation before stationary pattern formation in the feedback-absent system. Similar phenomenon appears in the reaction-diffusion model of the CO oxidation reaction on a Pt(110) single-crystal surface with global delayed feedback [8]. Their intrinsic connection also lies in the adjustment effect of the external delayed feedback on the intrinsic nonlinear feedback mechanism. Both of the peak's periodicities appearing in the diagrams are directly related to the original oscillation periodicity in feedback-absent system. When the external feedback that weakens the intrinsic nonlinear feedback (τ is at proper value) is strong enough, the original stable state is inhibited, and new phenomenon appears. It is interesting that similar situations also occur in the discrete cases of delayed coupled oscillators [21–23]. Because delay influences the system by adjusting the intrinsic nonlinear mechanism, the similar responses may reveal the similar nonlinear essences of the corresponding states. That is to say, between spatially extended systems and systems of coupled oscillators, Turing pattern may be analogous to the antiphase oscillations in nature while the bulk oscillation corresponds to the in-phase synchronization. In a recent paper [24], Vanag and Epstein have also pointed out the analogy between antiphase oscillations and Turing patterns when investigating oscillations in two coupled droplets.

When choosing other spots' or global averaged concentration as the detected signal, we find the results are of the same character. The situations under other parameter settings in Eq. (1) are also investigated. If the parameters are chosen in the region where the feedback-absent system can produce a Turing pattern, the results are also similar. If the parameters are chosen in the region where the feedback-absent system can produce bulk oscillation, the Turing pattern could be induced at some delay time when feedback is strong enough. This phenomenon is analogous to the main result of Ref. [8], in which the authors find spatiotemporal pattern could be induced by global delayed feedback in a surface chemical reaction. When the parameters are chosen in the region where the feedback-absent system can produce a uniform steady state, feedback is found to have no effect on the evolution process. In this situation, the transient oscillation process is so instantaneous that the effect of feedback is limited to a very narrow scope. Further investigations show the abovementioned result will vary under different initial conditions. When uniform steady state is used as initial condition, the results are generally of the same character, but the effect of feedback will change much depending on the exact initial value. When Turing pattern is employed as initial condition, the system will present stationary Turing after the readjustment of very weak transient oscillation, and the effect of feedback on PFT is also too limited to be detected.

IV. SUMMARY

The formation time control and pattern suppression phenomena of the Turing pattern are investigated in the delayed feedback controlled CDIMA reaction diffusion system. Numerical simulation results show that Turing pattern formation time can be prominently controlled if we adjust any one of the three factors, feedback intensity, delay time or feedback-imposing time. Turing pattern could also be suppressed or induced at proper parameter settings when feedback intensity is high enough. The striking similarity between bifurcation diagrams of the delayed feedback induced Turing pattern suppression and delay-induced bistability in discrete cases of delayed coupled oscillators indicates that Turing pattern may be analogous to antiphase oscillation in nature. These results not only provide an effective way to control the stationary pattern formation, but also present an important insight into the nature of the Turing pattern.

ACKNOWLEDGMENTS

This work was supported by the specialized research fund for the doctoral program of higher education (Grant No. 20020007027).

[1] M. Braune and H. Engel, Phys. Rev. E **62**, 5986 (2000).
[2] V. Petrov, S. Metens, P. Borckmans, G. Dewel, and K. Showalter, Phys. Rev. Lett. **75**, 2859 (1995).
[3] Th. Pierre, G. Bonhomme, and A. Atipo, Phys. Rev. Lett. **76**, 2290 (1996).
[4] W. Lu, D. Yu, and R. G. Harrison, Phys. Rev. Lett. **76**, 3316 (1996).
[5] G. Franceschini, S. Bose, and E. Schöll, Phys. Rev. E **60**, 5426 (1999).
[6] J. Tang and H. H. Bau, Phys. Rev. Lett. **70**, 1795 (1993).
[7] M. Bertram and A. S. Mikhailov, Phys. Rev. E **67**, 036207 (2003).
[8] M. Bertram and A. S. Mikhailov, Phys. Rev. E **63**, 066102 (2001).
[9] L. Yang, M. Dolnik, A. M. Zhabotinsky, and I. R. Epstein, Phys. Rev. E **62**, 6414 (2000).
[10] I. Lengyel and I. R. Epstein, Proc. Natl. Acad. Sci. U.S.A. **89**, 3977 (1992).
[11] B. Rudovics, E. Barillot, P. W. Davies, E. Dulos, J. Boissonade, and P. De Kepper, J. Phys. Chem. A **103**, 1790 (1999).
[12] F. Fecher, P. Strasser, M. Eiswirth, F. W. Schneider, and A. F. Münster, Chem. Phys. Lett. **313**, 205 (1999).
[13] A. K. Horváth, M. Dolnik, A. P. Muñuzuri, A. M. Zhabotinsky, and I. R. Epstein, Phys. Rev. Lett. **83**, 2950 (1999).
[14] M. Dolnik, A. M. Zhabotinsky, and I. R. Epstein, Phys. Rev. E **63**, 026101 (2001).
[15] A. Sanz-Anchelergues, A. M. Zhabotinsky, I. R. Epstein, and A. P. Muñuzuri, Phys. Rev. E **63**, 056124 (2001).
[16] M. Dolnik, I. Berenstein, A. M. Zhabotinsky, and I. R. Epstein, Phys. Rev. Lett. **87**, 238301 (2001).
[17] S. Rudiger, D. G. MÍguez, A. P . Muñuzuri, F. Sagués, and J. Casademunt, Phys. Rev. Lett. **90**, 128301 (2003).
[18] I. Lengyel and I. R. Epstein, Science **251**, 650 (1991).
[19] A. P. Muñuzuri, M. Dolnik, A. M. Zhabotinsky, and I. R. Epstein, J. Am. Chem. Soc. **121**, 8065 (1999).
[20] I. Lengyel, G. Rábai, and I. R. Epstein, J. Am. Chem. Soc. **112**, 9104 (1990).
[21] G. Kozyreff, A. G. Vladimirov, and P. Mandel, Phys. Rev. Lett. **85**, 3809 (2000).
[22] G. Kozyreff, A. G. Vladimirov, and P. Mandel, Phys. Rev. E **64**, 016613 (2001).
[23] M. K. S. Yeung and S. H. Strogatz, Phys. Rev. Lett. **82**, 648 (1999).
[24] V. K. Vanag and I. R. Epstein, J. Chem. Phys. **119**, 7297 (2003).